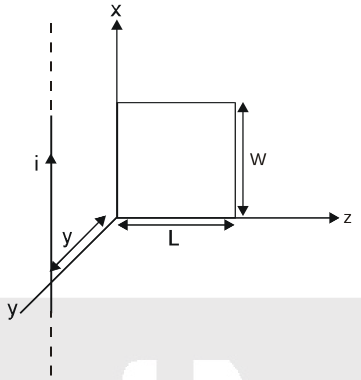
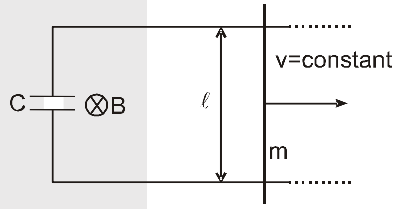
High Level Problems (HLP)

SUBJECTIVE QUESTIONS

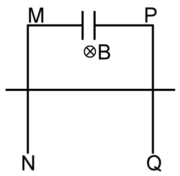
1. In the figure, a long thin wire carrying a varying current $i = i_0 \sin \omega t$ lies at a distance y above one edge of a rectangular wire loop of length L and width W lying in the x-z plane. What emf is induced in the loop.



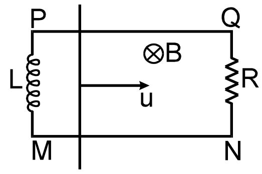
2. In the figure shown a conducting rod of length ℓ, resistance R and mass m is moved with a constant velocity v. The magnetic field B varies with time t as B = 5 t, where t is time in second. At t = 0 the area of the loop containing capacitor and the rod is zero and the capacitor is uncharged. The rod started moving at t = 0 on the fixed smooth conducting rails which have negligible resistance. Find the current in the circuit as a function of time t.



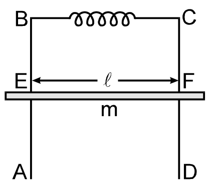
3. In the figure shown a conducting rod of length ℓ, resistance R & mass m can move vertically downward due to gravity. Other parts are kept fixed. B = constant = B_0. MN and PQ are vertical, smooth, conducting rails. The capacitance of the capacitor is C. The rod is released from rest. Find the maximum current in the circuit.



4. In the figure, a conducting rod of length ℓ = 1 meter and mass m = 1 kg moves with initial velocity u = 5 m/s. on a fixed horizontal frame containing inductor L = 2 H and resistance R = 1 Ω. PQ and MN are smooth, conducting wires. There is a uniform magnetic field of strength B = 1T. Initially there is no current in the inductor. Find the total charge in coulomb, flown through the inductor by the time velocity of rod becomes v_f = 1 m/s and the rod has travelled a distance x = 3 meter.

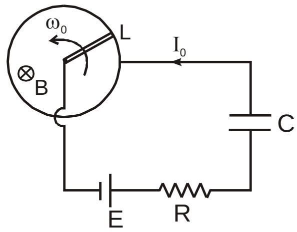


5. A conducting frame ABCD is kept fixed in a vertical plane. A conducting rod EF of mass m can slide smoothly on it remaining horizontal always. The resistance of the loop is negligible and inductance is constant having value L. The rod is left from rest and allowed to fall under gravity and inductor has no initial current. A uniform magnetic field of magnitude B is present throughout the loop pointing inwards. Determine.

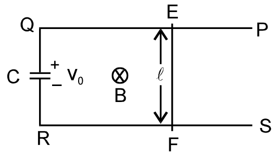


(a) position of the rod as a function of time assuming initial position of the rod to be x = 0 and vertically downward as the positive X-axis.
(b) maximum current in the circuit
(c) maximum velocity of the rod.

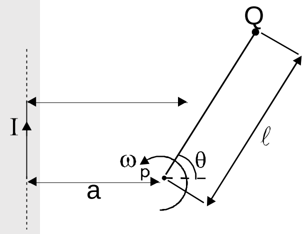
6. L is s smooth conducting loop of radius ℓ = 1.0 m & fixed in a horizontal plane. A conducting rod of mass m = 1.0 kg and length slightly greater than ℓ hinged at the centre of the loop can rotate in the horizontal plane such that the free end slides on the rim of the loop. There is a uniform magnetic field of strength B = 1.0 T directed vertically downward. The rod is rotated with angular velocity ω_0 = 1.0 rad/s and left. The fixed end of the rod and the rim of the loop are connected through a battery of e.m.f. E, a resistor of resistance R = 1.0 Ω, and initially uncharged capacitor of capacitance C = 1.0 F in series. Find :
(i) the time dependence of e.m.f. E such that the current I_0 = 1.0 A in the circuit is constant.
(ii) energy supplied by the battery by the time rod stops .



7. In the figure shown 'PQRS' is a fixed resistanceless conducting frame in a uniform and constant magnetic field of strength B. A rod 'EF' of mass 'm', length 'ℓ' and resistance R can smoothly move on this frame. A capacitor charged to a potential difference 'V_0' initially is connected as shown in the figure. Find the velocity of the rod as function of time 't' if it is released at t = 0 from rest.



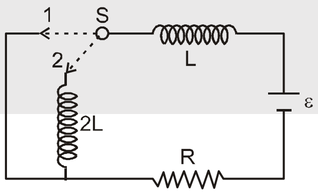
8. In the figure shown a long conductor carries constant current I. A rod PQ of length ℓ is in the plane of the rod. The rod is rotated about point P with constant angular velocity ω as shown in the figure. Find the e.m.f. induced in the rod in the position shown. Indicate which point is at high potential.



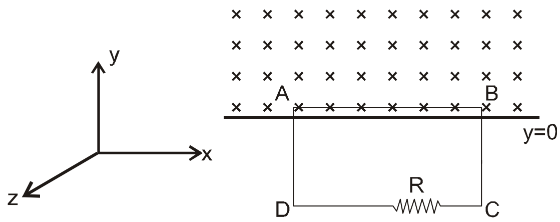
9. An infinitesimally small bar magnet of dipole moment M is moving with the speed v in the X-direction. A small closed circular conducting loop of radius 'a' and negligible self-inductance lies is the Y-Z plane with its centre at x = 0, and its axis coinciding with the X-axis. Find the force opposing the motion of the magnet, if the resistance of the loop is R. Assume that the distance x of the magnet from the centre of the loop is much greater than a.

10. A square loop of side a = 12 cm with its sides parallel to x, and y-axis is moved with velocity, V = 8 cm/s in the positive x direction in a magnetic field along the positive z-direction. The field is neither uniform in space nor constant in time. It has a gradient $\partial B/\partial x = -10^{-3}$ T/cm along the x-direction, and it is changing in time at the rate $\partial B/\partial t$ = 7T/sec in the loop if its resistance is R = 4.5 Ω. Find the current.

11. In the circuit shown, the switch S is shifted to position 2 from position 1 at t = 0, having been in position 1 for a long time. Find the current in the circuit as a function of time.



12. A square loop ABCD of side ℓ is moving in xy plane with velocity $\vec{v} = \beta t\ \hat{j}$. There exists a non-uniform magnetic field $\vec{B} = -B_0(1 + \alpha y^2)\ \hat{k}$ (y > 0), where B_0 and α are positive constants.Initially, the upper wire of the loop is at y = 0. Find the induced voltage across the resistance R as a function of time. Neglect the magnetic force due to induced current.

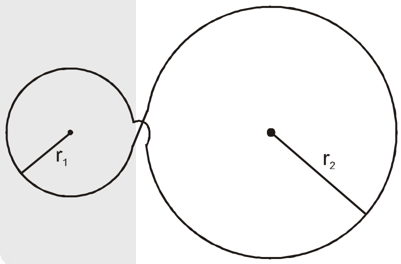


13. A thin wire ring of radius a and resistance r is located inside a long solenoid so that their axes coincide. The length of the solenoid is equal to ℓ, its cross-sectional radius, to b. At a certain moment the solenoid was connected to a source of a constant voltage V. The total resistance of the circuit is equal to R. Assuming the inductance of the ring to be negligible, find the maximum value of the radial force acting per unit length of the ring.

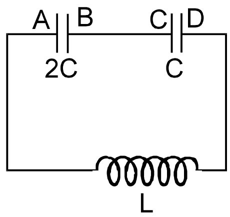
14. A long cylinder of radius a carrying a uniform surface charge rotates about its axis with an angular velocity ω. Find the magnetic field energy per unit length of the cylinder if the linear charge density equals λ and $\mu_r = 1$.

15. A long solenoid of length ℓ = 2.0m, radius r = 0.1m and total number of turns N = 1000 is carrying a current i_0 = 20.0A. The axis of the solenoid coincides with the z-axis.
(a) State the expression for the magnetic field of the solenoid and calculate its value?
Magnetic field
(b) Obtain the expression for the self-inductance (L) of the solenoid. Calculate its value.
Value of L
(c) Calculate the energy stored (E) when the solenoid carries this current?
(d) Let the resistance of the solenoid be R. It is connected to a battery of emf e. Obtain the expression for the current (i) in the solenoid.
(e) Let the solenoid with resistance R described in part (d) be stretched at a constant speed υ (ℓ is increased but N and γ are constant). State Kirchhoff's second law for this case. (Note: Do not solve for the current.)
(f) Consider a time varying current $i = i_0 \cos(\omega t)$ (where i_0 = 20.0A) flowing in the solenoid. Obtain an expression for the electric field due to the current in the solenoid. (Note: Part (e) is not operative, i.e. the solenoid is not being stretched.)
(g) Consider $t = \pi/2\omega$ and $\omega = 200/\pi$ rad–s^{-1} in the previous part. Plot the magnitude of the electric field as a function of the radial distance from the solenoid. Also, sketch the electric lines of force.

16. The wire loop shown in the figure lies in uniform magnetic induction $B = B_0 \cos \omega t$ perpendicular to its plane. (Given r_1 = 10 cm and r_2 = 20 cm, B_0 = 20 mT and $\omega = 100\,\pi$). Find the amplitude of the current induced in the loop if its resistance is 0.1 Ω/m.



17. Two capacitors of capacitances 2C and C are connected in series with an inductor of inductance L. Initially capacitors have charge such that $V_B - V_A = 4V_0$ and $V_c – V_D = V_0$. Initial current in the circuit is zero. Find:
(a) Maximum current that will flow in the circuit.
(b) Potential difference across each capacitor at that instant.
(c) equation of current flowing towards left in the inductor.



HLP Answers

1. $\frac{\mu_0 i_0 W\omega\cos\omega t}{4\pi}\ell n\left(\frac{L^2}{Y^2}+1\right)$

2. $i = 10\,\ell\,v\,c\,(1 - e^{-t/Rc})$

3. $i_{max} = \frac{mg\,B\,\ell\,c}{m + B^2\,\ell^2\,c}$

4. $Q = \frac{-\frac{B^2\,\ell^2}{R}\,x - m\,(v_f - u)}{B\,\ell} = 1C.$

5. (a) $x = \frac{g}{\omega^2}[1-\cos\omega t]$, (b) $I_{max} = \frac{2mg}{B\ell}$, (c) $V_{max} = \frac{g}{\omega}$

6. **(i)** $\frac{1}{2}+\frac{7t}{4}$ (ii) $\frac{13}{18}$ J

7. $v = \frac{B\ell C\;V_0}{m+B^2\ell^2C}\left(1 - e^{-\left(\frac{B^2\,\ell^2}{mR}+\frac{1}{RC}\right)t}\right)$

8. $\frac{\mu_0 i\omega}{2\pi\cos\theta}\left[\ell - \frac{a}{\cos\theta}\ell n\left(\frac{a+\ell\cos\theta}{a}\right)\right]$

9. $\frac{9\mu_0^{\,2}M^2a^4v}{4\;Rx^8}$

10. 22.4 mA

11. $I = \frac{\varepsilon}{R}\left(1-\frac{2}{3}\times e^{\frac{-Rt}{3L}}\right)$

12. $\varepsilon = -B_0 I\beta\left(t+\frac{\alpha\times\beta^2t^5}{4}\right)$

13. $\frac{\mu_0 a^2V^2}{4rRIb^2}$

14. $\frac{\mu_0 a^2\omega^2\lambda^2}{8\pi}$

16. π ampere.

17. (a) $I_{max} = \left(\sqrt{\frac{6C}{L}}\right)v_0$; (b) $3v_0$, $3v_0$; (c) $i = I_{max}\sin\omega t$; $\omega = \left(\sqrt{\frac{3}{2LC}}\right)$